E23.23 [1VG] Note:exercise 4, written exam, June 23th, 2012.

Exercises

A function $f(x) = \sum_{n=0}^{\infty} a_n x_n$, analytic in a neighborhood of 0, satisfies on its domain the conditions

$$\begin{cases} f'(x) = 1 + f(-x) \\ f(0) = c \end{cases}$$
 (note that this is not a Cauchy problem!).

Determine *f*.
Prove that the function found is the only solution, in the set of all functions that can be derived in a neighborhood of 0.