4 Natural numbers [1X9]

We want to properly define the set

$$\mathbb{N} = \{0, 1, 2, \ldots\}$$

of the natural numbers.

A possible model, as shown in Sec. [246], is obtained by relying on the theory of Zermelo—Fraenkel.

Here instead we present Peano's axioms, expressed using the *naive version* of set theory.

Definition 4.1 (Peano's axioms)**.** *[1XB]*

From those two important properties immediately follow. One is the principle of induction, see [1XC]. The other is left for exercise.

Exercise 4.2. *[1YP]*

The idea is that the successor function encodes the usual numbers according to the scheme

$$1 = S(0), \quad 2 = S(1), \quad 3 = S(2) \ldots$$

and (having defined the addition) we will have that $S(n) = n + 1$.

Exercise 4.3. *[1XD]*

4.1 Induction

[27J]

4.2 Recursive definitions

[274]

4.3 Arithmetic

[0NN]

4.4 Ordering

[27K]

4.5 Z-F and Peano compatibility

[26F]

4.6 Generalized induction, well ordering

[27M]

4.7 Frequently, eventually

[26G]