Definition 4.2. [1XB] *(Solved on 2022-11-03)*

(N1) *There is a number* $0 \in \mathbb{N}$.

(N2) *There is a function* $S : \mathbb{N} \to \mathbb{N}$ *(called "successor"), such that* [a]

(N3) $S(x) \neq 0$ *for each* $x \in \mathbb{N}$ *and*

(N4) S *is injective, that is,* $x \neq y$ *implies* $S(x) \neq S(y)$.

(N5) *If* U *is a subset of* $\mathbb{N}$ *such that:* $0 \in U$ *and* $\forall x, x \in U \Rightarrow S(x) \in U$ *, then* $U = \mathbb{N}$.

We will often write Sn *instead* $S(n)$ *to ease notations.*

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[a] We are using the same word *successor* used in the definition [1Z0] for well ordered sets, and in [24X] in Zermelo-Fraenkel theory: we will see how these definition are "compatible".