

Proposition 4.a.2. [1XC] Let $A \supseteq \mathbb{N}$ and $P(n)$ be a logical proposition that can be evaluated for $n \in A$. Suppose the following two assumptions are satisfied:

- $P(n)$ is true for $n = 0$ and
- $\forall n \in \mathbb{N}, P(n) \Rightarrow P(S(n))$;

then P is true for every $n \in \mathbb{N}$.

Proof. Let $U = \{n \in \mathbb{N} : P(n)\}$, we know that $0 \in U$ and that $\forall x, x \in U \Rightarrow S(x) \in U$, then from (N5) we conclude that $U = \mathbb{N}$. \square