Exercise 4.5. [1XG] *Prove* [a] *by induction the following assertions:*

1. $\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$;
2. $\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$;
3. $\sum_{k=1}^{n} k^3 = \frac{n^2(n+1)^2}{4}$;
4. $\sum_{k=1}^{n} \frac{1}{4k^2-1} = \frac{n}{2n+1}$;
5. $\sum_{k=1}^{n} \frac{k}{2^k} = 2 - \frac{n+2}{2^n}$;
6. $n! \geq 2^{n-1}$;
7. *If* $x > -1$ *is a real number and* $n \in \mathbb{N}$ *then* $(1+x)^n \geq 1 + nx$ *(Bernoulli inequality).*

Solution 1. [1XK]

[a] In the following exercises we give for good knowledge of the operations typical of the natural numbers, and their order relation.