Exercise 4.51. [1XY] Difficulty:*. *Let A be a well-ordered set* [a] *by the order $\leq$; let $m = \min A$; then for propositions $P(a)$ with $a \in A$ you can use a proof method, called* transfinite induction, *in which*

- *$P(m)$ is required to be true, and*
- *the following "inductive step" is proven:*

$$\forall n \in \mathbb{N}\Big(\big(\forall k < n, P(k)\big) \Rightarrow P(n)\Big)$$

Show that if the proposition P satisfies the previous two requirements, then $\forall x \in A, P(x)$.
Prove also that if $A = \mathbb{N}$ then the "inductive step" is equivalent to the inductive step of strong induction (defined in [1XS]*).*

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[a] As defined in [07R].