

Definition 3.52. [1Y0] *The axiom of specification, which reads*

If A is a set, and $P(x)$ is a logical proposition, then $\{x \in A : P(x)\}$ is a set.

Formally, setting $B = \{x \in A : P(x)\}$,

$$\forall X, X \in B \iff X \in A \wedge P(x) \quad .$$

This axiom avoids Russell's paradox: let A be the set of x such that $x \notin x$, then you have neither $A \in A$ nor $A \notin A$.