Definition 3.52. [1Y1] *The **axiom of the power set** says that for every set A, there is a set $\mathcal{P}(A)$ whose elements are all and only subsets of A. A shortened definition formula is*

$$\mathcal{P}(A) \stackrel{def}{=} \{B : B \subseteq A\} \quad .$$

$\mathcal{P}(A)$ is also called set of parts.
In the formal language of the Zermelo-Fraenkel axioms, the axiom is written:

$$\forall A, \exists Z, \forall y, y \in Z \iff (\forall z, z \in y \implies z \in A) \quad ;$$

this formula implies that the power set Z is unique, therefore we can denote it with the symbol $\mathcal{P}(A)$ without fear of misunderstandings.
Note that

$$(\forall z, z \in y \implies z \in A)$$

can be shortened with $y \subseteq A$ and therefore the axiom can be written as

$$\forall A, \exists Z, \forall y, y \in Z \iff (y \subseteq A) \quad ;$$

then using the extensionality, we obtain that

$$Z = \{y : (y \subseteq A)\} \quad .$$