

Exercise 3.ii.35. [1Y9] Let be given a, b, x, y .

1. Show that in the hypothesis

$$\{a, b\} = \{x, y\}$$

you have that

$$(a = b) \iff (x = y) \iff a = b = x = y \quad .$$

2. In particular, you deduce that if

$$\{a\} = \{x, y\}$$

then $a = x = y$.

3. Then show that if we assume that the four elements a, b, x, y are not all the same, then we have

$$\{a, b\} = \{x, y\}$$

if and only if $a = x \wedge b = y$ or $a = y \wedge b = x$.

To show the above be as precise as possible: use the axiom of extensionality [1Y8], the axiom of pairing [1Y3] and the tautologies shown in the previous section (or other elementary logical relationships).

Solution 1. [1YB]