**Exercise 3.172.** [1YM]Prerequisites:[01R],[(3.171)],[24V].[a] *Prove that*

$$x = y \iff S(x) = S(y) \quad .$$

*In particular this shows that, if $A$ is an $S$-saturated set, then the function $S : A \to A$ is well defined, and its graph is the relation*

$$\{(x, y) \in A^2 : y = S(x)\} \quad ;$$

*moreover $S$ is injective.*

**Solution 1.** [1YN]

[a] Proposition 1.7.4 point 5 in [3].