§2.a Logic [1YS]

[23H]

§2.a.a Propositions

Definition 2.1. *[1VW]*

Example 2.2. *[1VX]*

[23J]
A proposition may depend on some variables. Examples:

- ”the person x by trade is a baker”,
- ”the number x is greater than 9”.

We write

$$P(x) \doteq \text{“the number x is larger than 9'}$$

to say that $P(x)$ is the symbol that summarizes the proposition written on the right.

Remark 2.3. *[23K]*

§2.a.b Propositional logic

Definition 2.4. *[00D]*

[[00F]]

Definition 2.5. *[00G]*

[1YK]

Remark 2.6. *[228]*

[[00H]]

Definition 2.7. *[00J]*

So a well-formed formula is a ”*logical proposition*” as it takes on the value of truth or falsehood, depending on the value given to its free variables. We can broaden the definition by adding that the propositions seen in the previous section they are ”atomic formulas”; For example,

$$\text{”}x \text{ is a number less than 3”} \wedge \text{”}y \text{ is an even number”}$$

it will also be a ”well-formed formula”.

For convenience, in this Section, we also add to the language the constants V and F which are respectively always true and always false, in every evaluation. †5 In the construction of well-formed formulas they are treated as variables. Note that we have not introduced the equality connective ”=”. When all variables can only take true/false values, the equality $a = b$ can be interpreted as $a \iff b$. In more general contexts (as in the case of set theory) instead, ”equality” needs a precise definition.

†5 We can get rid of constants V and F by defining them as $V = A \vee \neg A$ and $F = \neg V$.

Exercises

E2.8 [1VY]

E2.9 [00K]

E2.10 [00N]

E2.11 [22C]

E2.12 [2G8]

§2.a.c First-order logic

In the first order logic we add the connectives ∀, which reads *"for each"* and ∃, which reads *"exists"*. We must therefore enlarge the family of **well-formed formulas**.

Definition 2.13. *[00Q]*

In every part of a formula where a variable is quantified this variable can be replaced with every other variable.

Remark 2.14. *[1X1]*

Remark 2.15. *[2DC]*

Note that, in many examples, quantified variables are assumed to be elements of a "set".

Definition 2.16. *[1X2]*

Definition 2.17. *[00R]*

We use the term "together" informally here, see footnote [01J].

Remark 2.18. *[00S]*

[2GV]

Since an element of a set may not have a truth/falsehood value, we enrich the language by adding the "*logical propositions*".

Definition 2.19. *[00T]*

An example of a logical proposition would be: *"n is an even number"*. We can use logical propositions as atoms in the construction of well-formed formulas.

Exercises

E2.20 [00V]

E2.21 [00X]

E2.22 [00Z]

E2.23 [011]

E2.24 [016]

E2.25 [013]