

### §3.j Cardinality

[1YW]

[22B]

**Proposition 3.j.1.** [1Z9]

In the following, let  $E_0 = \emptyset$ , and let  $E_n = \{1, \dots, n\}$  otherwise if  $n \geq 1$ .

**Lemma 3.j.2.** [2GK]**Definition 3.j.3.** [1B1]

Note that the null map  $f : \emptyset \rightarrow \emptyset$  is a bijection; and  $|A| = 0 \Leftrightarrow A = \emptyset$ . The following exercise is a fundamental result.

**Exercise 3.j.4.** [2GH]

We recall Theorem 1.12.2 of the notes [3], for convenience.

**Theorem 3.j.5.** [02S]**Definition 3.j.6.** [2DD]

#### §3.j.a Finite sets

##### Exercises

E3.j.7 [02T]

E3.j.8 [02W]

E3.j.9 [02Y]

E3.j.10 [22K]

#### §3.j.b Comparison

##### Exercises

E3.j.11 [030]

E3.j.12 [031]

E3.j.13 [032]

E3.j.14 [034]

E3.j.15 [036]

E3.j.16 [038]

E3.j.17 [03C]

E3.j.18 [03F]

#### §3.j.c Countable cardinality

**Definition 3.j.19.** [2DF]

**Exercises**

E3.j.20 [03H]

E3.j.21 [03M]

E3.j.22 [03P]

E3.j.23 [03R]

**§3.j.d Cardinality of the continuum****Definition 3.j.24.** [03V]**Remark 3.j.25.** [2F2]

[ [03W] ]

**Exercises**

E3.j.26 [03X]

E3.j.27 [03Y]

E3.j.28 [040]

E3.j.29 [043]

E3.j.30 [045]

**§3.j.e In general**

Let's add some more general exercises.

**Exercises**

E3.j.31 [048]

E3.j.32 [04B]

E3.j.33 [04D]

E3.j.34 [04G]

E3.j.35 [04J]

E3.j.36 [22M]

E3.j.37 [04M]

E3.j.38 [04P]

E3.j.39 [04R]

E3.j.40 [04V]

E3.j.41 [04X]

E3.j.42 [04Z]

**Remark 3.j.43.** [27H]

**Exercises**

E3.j.44 [051]

E3.j.45 [053]

E3.j.46 [055]

E3.j.47 [057]

Other interesting exercises are [0T9], [0MZ].

**QuasiEsercizio 13.** [1ZB]**QuasiEsercizio 14.** [05B]**QuasiEsercizio 15.** [05C]**QuasiEsercizio 16.** [05F]**QuasiEsercizio 17.** [05G]**§3.j.f Power**

Recall that  $A^B$  is the set of all functions  $f : B \rightarrow A$ . We will write  $|2^A|$  to indicate the cardinality of the set of parts of  $A$ .

**Exercises**

E3.j.48 [05J]

E3.j.49 [05M]

In general in case  $|B| > |A|$  the study of the cardinality of  $|B^A|$  is very complex (even in seemingly simple cases like  $A = \mathbb{N}$ ).

[ [05Q] ]