

Exercise 3.216. [1Z1] Prerequisites: [1Z0]. (Proposed on 2023-01-17)

Suppose X has no maximum; let S be defined as in [1Z0]; show that is an injective function

$$S : X \rightarrow X \quad ,$$

and that $S(x) \neq 0_X$, for every x (that is, 0_X is not successor of any element).

Solution 1. [223]

We note that in general S will not be surjective, as a function $S : X \rightarrow X \setminus \{0_X\}$: there may be elements $y \in S, y \neq 0_X$ that are not successors of an element. If, however, for a given $y \in X$, there exists $x \in X$ such that $y = S(x)$, we will say that x is the **predecessor** of y .