Proposition 3.167. [1Z6]

- *Suppose that the function* $f : A \times A \to B$ *is invariant for the equivalence relation* $\sim$ *in all its variables, i.e.*

  $$\forall x, y, v, w \in A, \quad x \sim y \wedge v \sim w \Rightarrow f(x, v) = f(y, w) \quad ;$$

  let $\tilde{f}$ *be the projection to the quotient* $\widetilde{f} : {}^{A}/_{\sim} \times {}^{A}/_{\sim} \to B$ *that satisfies*

  $$f(x, y) = \widetilde{f}(\pi(x), \pi(y)) \quad .$$

  If f *is commutative* (resp. *associative) then* $\widetilde{f}$ *is commutative* (resp. *associative).*

- *If* R *is a relation in* $A \times A$ *invariant for* $\sim$*, and* R *is reflexive* (resp *symmetrical, antisymmetric, transitive) then* $\widetilde{R}$ *is reflexive* (resp *symmetrical, antisymmetric, transitive).*

- *If* A *and* B *are ordered and the order is invariant, and* f *is monotonic, then* $\widetilde{f}$ *is monotonic.*