Proposition 2.196. [1Z6]

- *Suppose that the function* $f : A \times A \to A$ *is invariant for the equivalence relation* $\sim$ *in all its variables, in the sense defined in* [(2.194)] *let* $\tilde{f}$ *be the projection to the quotient*

$$\tilde{f} : {}^{A}/_{\sim} \times {}^{A}/_{\sim} \to {}^{A}/_{\sim} \quad .$$

  If f *is commutative (*resp. *associative) then* $\tilde{f}$ *is commutative (*resp. *associative).*

- *If* R *is a relation in* $A \times A$ *invariant for* $\sim$*, and* R *is reflexive (*resp *symmetrical, antisymmetric, transitive) then* $\tilde{R}$ *is reflexive (*resp *symmetrical, antisymmetric, transitive).*

- *Consider the ordered sets* $(A, \leq_A)$ *and* $(B, \leq_B)$*, let* $f : A \to B$ *be a monotonic function; suppose moreover that* $\leq_A$ *is invariant with respect to an equivalence relation* $\sim$ *on* A*, e and let* $\tilde{f} : {}^{A}/_{\sim} \to B$ *be its projection to the quotient: then* $\tilde{f}$ *is monotonic.*