Proposition 3.194. *[1Z7]* (Replaces 06G) (Replaces 06H) *Consider R a transitive and reflexive relation in $A \times A$; such a relation is called a* preorder *[**?**]; we define $x \sim y \iff (xRy \wedge yRx)$ then $\sim$ is an equivalence relation, R is invariant for $\sim$, and $\widetilde{R}$ (defined as in [1Z6]) is an order relation.*

Proof. 1. $\sim$ is clearly reflexive and symmetrical; is transitive because if $x \sim y, y \sim z$ then $xRy \wedge yRx \wedge yRz \wedge zRy$ but being R transitive you get $xRz \wedge zRx$ i.e. $x \sim z$

2. Let $x, y, \tilde{x}, \tilde{y} \in X$ be such that $x \sim \tilde{x}, y \sim \tilde{y}$ then we have $xR\tilde{x} \wedge \tilde{x}Rx \wedge yR\tilde{y} \wedge \tilde{y}Ry$ if we add xRy, by transitivity we get $\tilde{x}R\tilde{y}$; and symmetrically.

3. Finally, we see that $\widetilde{R}$ is an order relation on Y. Using the (well posed) definition "$[x]\widetilde{R}[y] \iff xRy$" we deduce that $\widetilde{R}$ is reflexive and transitive (as indeed stated in the previous proposition). $\widetilde{R}$ is also antisymmetric because if for $z, w \in A/\sim$ you have $z\widetilde{R}w \wedge w\widetilde{R}z$ then taken $x \in z, y \in w$ we have $xRy \wedge yRx$ which means $x \sim y$ and therefore $z = w$.

□