Exercises

E2.193 [1Z8] $\mathbb{Z}$ are the relative integers with the usual operations. Let $p \geq 1$ a fixed integer. Consider the equivalence relation

$$n \sim m \iff p|(n-m)$$

that is, they are equivalent when $n - m$ is divisible by p.

Show that there are p equivalence classes $[0], [1], \ldots [p-1]$ We indicate the quotient space with $\mathbb{Z}/(p\mathbb{Z})$ or more briefly $\mathbb{Z}_p$.

Show that the usual operations of sum and product in $\mathbb{Z}$ are invariant (in the sense defined in [(2.194)]), hence they pass to the quotient.