5   Groups, Rings, Fields [1ZD]

We review these definitions.

Definition 5.1. [1ZF]

Definition 5.2. [1ZG]

We assume that $0 \neq 1$ (otherwise $\{0\}$ would be a ring).

Examples of commutative rings are: integer numbers $\mathbb{Z}$, polynomials $A[x]$ with coefficients in a commutative ring A.

An example of a non-commutative ring is given by matrixes $\mathbb{R}^{n \times n}$, with the usual operation of multiplication and addition.

Definition 5.3. [1ZH]

Some field examples are: rational numbers $\mathbb{Q}$, the real numbers $\mathbb{R}$ and the complex numbers $\mathbb{C}$.

Remark 5.4. [20R]

Remark 5.5. [1ZW]

Definition 5.6. [1ZJ]

Examples of ordered field are: rational numbers $\mathbb{Q}$ the real numbers $\mathbb{R}$. The complex numbers $\mathbb{C}$ do not allow an ordering satisfying the above properties (see exercise [08V]).

Definition 5.7. [1ZK]

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Exercises

E5.8 [1ZM]

E5.9 [1ZP]

E5.10 [29C]

E5.11 [1ZR]

E5.12 [1ZS]

E5.13 [203]

E5.14 [1ZT]

E5.15 [1ZV]

E5.16 [1ZX]

E5.17 [1ZY]

E5.18 [1ZZ]

E5.19 [08V]

E5.20 [200]

E5.21 [202]

E5.22 [20T]

E5.23 [205]

QuasiEsercizio 19. [08W]

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[38]Parts of the following exercises are from Chap. 2 Sec. 2 in [3], or Chap. 1 in [19].