

**Definition 5.2.** [1ZF] (Solved on 2022-11-15) A **group** is a set  $G$  equipped with a binary operation  $*$ , that associates an element  $a * b \in G$  to each pair  $a, b \in G$ , respecting these properties.

1. *Associative property: for any given  $a, b, c \in G$  we have  $(a * b) * c = a * (b * c)$ .*
2. *Existence of the neutral element: an element denoted by  $e$  such that  $a * e = e * a = a$ .*
3. *Existence of the inverse: each element  $a \in G$  is associated with an **inverse element**  $a'$ , such that  $a * a' = a' * a = e$ . The inverse of the element  $a$  is often denoted by  $a^{-1}$  (or  $-a$  if the group is commutative).<sup>a</sup>*

A group is said to be **commutative** (or **abelian**) if moreover  $a * b = b * a$  for each pair  $a, b \in G$ .

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<sup>a</sup>The notation  $a^{-1}$  is justified by the fact that the inverse element is unique: cf [1ZP].