Definition 5.7. [1ZJ] *An **ordered ring** F is a ring with a total order relation $\leq$ for which, for every $x, y, z \in F$,*

- $x \leq y \Rightarrow x + z \leq y + z$;

- $x, y \geq 0 \Rightarrow x \cdot y \geq 0$.

Due to [203], if F is a field, in the second hypothesis we may equivalently write $x, y > 0 \Rightarrow x \cdot y > 0$. (Regarding the second hypothesis, see also [1ZT]) For further informations see the references in [29]. We will assume that in an ordered ring the multiplication is commutative.