

Exercises

E5.15 [1ZT] Suppose that in a ring A there is a total ordering \leq such that for every $x, y, z \in A$ you have $x \leq y \Rightarrow x + z \leq y + z$; then show that these are equivalent

$$\bullet \quad x \leq y \wedge 0 \leq z \quad \Rightarrow \quad x \cdot z \leq y \cdot z;$$

$$\bullet \quad x \geq 0 \wedge y \geq 0 \quad \Rightarrow \quad x \cdot y \geq 0.$$