Exercises

E5.16 [1ZV]Prerequisites:[29D],[1ZS],[1ZT]. Prove [a] than in an ordered ring F:

1. for each $x \in F, x^2 \geq 0$, in particular $1 = 1^2 > 0$;
2. $x > 0 \Rightarrow -x < 0$
3. $y > x \Rightarrow -y < -x$;
4. $x \leq y \wedge a \leq 0 \Rightarrow a \cdot x \geq a \cdot y$;
5. $x \geq a \wedge y \geq b \Rightarrow x + y \geq a + b$;
6. $x > a \wedge y \geq b \Rightarrow x + y > a + b$;
7. $x \geq a \geq 0 \wedge y \geq b \geq 0 \Rightarrow x \cdot y \geq a \cdot b$;

Prove than in an ordered field F:

1. $x > a > 0 \wedge y > b \geq 0 \Rightarrow x \cdot y > a \cdot b$;
2. $x > 0 \Rightarrow x^{-1} > 0$;
3. $y > x > 0 \Rightarrow x^{-1} > y^{-1} > 0$;
4. $x \cdot y > 0$ if and only if x and y agree on sign (i.e. either both > 0 or both < 0);

Solution 1. *[29B]*

[a] From Cap. 2 Sec. 7 in [3], or [20] Prop. 1.18