

Exercise 5.17. [1ZX] In an ordered field F we call $P = \{x \in F : x \geq 0\}$ the set of positive (or zero) numbers; it satisfies the following properties:

^a

- $x, y \in P \Rightarrow x + y, x \cdot y \in P,$
- $P \cap (-P) = \{0\}$ and
- $P \cup (-P) = F.$

vice versa if in a field F we can find a set $P \subseteq F$ that satisfies them, then F is an ordered field by defining $x \leq y \Leftrightarrow y - x \in P.$

^aFrom Chap. 2 Sect. 7 in [?]