

**Exercise 5.22.** [202] Let  $F$  be a field; given  $\alpha \neq 0$  and  $h \in \mathbb{N}$  consider the recursive definition of exponentiation  $\alpha^h$  defined from  $\alpha^0 = 1$  and  $\alpha^{(n+1)} = \alpha^n \cdot \alpha$ ; then prove that  $\alpha^{h+k} = \alpha^h \alpha^k$  and  $(\alpha^h)^k = \alpha^{(hk)}$  for every  $k, h \in \mathbb{N}$ .