Exercise 5.24. [205] *Let F be a commutative ring, $a, b \in F$, $n \in \mathbb{N}$ then*

$$(a+b)^n = \sum_{k=0}^{n} \binom{n}{k} a^{n-k} b^k$$

where the factor

$$\binom{n}{k} \stackrel{def}{=} \frac{n!}{k!(n-k)!}$$

is called the "binomial coefficient". *(This result is known as the binomial theorem, Newton's formula, Newton's binomial). To prove it by induction, check that*

$$\binom{n+1}{k+1} = \binom{n}{k+1} + \binom{n}{k}$$

for $0 \le k, k+1 \le n$.