Proposition 6.20. [208](Solved on 2022-11-24) *Let therefore $A \subseteq \mathbb{R}$ be not empty, let $l \in \mathbb{R} \cup \{+\infty\}$; you can easily demonstrate the following properties:*

$\sup A \le l$	$\forall x \in A, x \le l$
$\sup A > l$	$\exists x \in A, x > l$
$\sup A < l$	$\exists h < l, \forall x \in A, x \le h$
$\sup A \ge l$	$\forall h < l, \exists x \in A, x > h$

the first and third derive from the definition of supremum, [a] *the second and fourth by negation; in the third we can conclude equivalently that $x < h$, and in the fourth that $x \ge h$.*

If $l \neq +\infty$ then also we can also write (replacing $h = l - \varepsilon$)

$\sup A < l$	$\exists \varepsilon > 0, \forall x \in A, x \le l - \varepsilon$
$\sup A \ge l$	$\forall \varepsilon > 0, \exists x \in A, x > l - \varepsilon$

[a] In particular in the third you can think that $h = \sup A$.