

**Proposition 6.24.** [20B] Let  $A \subseteq \mathbb{R}$  not empty, let  $l \in \mathbb{R} \cup \{-\infty\}$ ; the following properties apply:

$\inf A \geq l$	$\forall x \in A, x \geq l$
$\inf A < l$	$\exists x \in A, x < l$
$\inf A > l$	$\exists h > l, \forall x \in A, x \geq h$
$\inf A \leq l$	$\forall h > l, \exists x \in A, x < h$

If  $l \neq -\infty$  then also we write (substituting  $h = l + \varepsilon$ )

$\inf A > l$	$\exists \varepsilon > 0, \forall x \in A, x \geq l + \varepsilon$
$\inf A \leq l$	$\forall \varepsilon > 0, \exists x \in A, x \leq l + \varepsilon$