Definition 6.38. *[20F]* *Let $I \subset \mathbb{R}$, $x_0 \in \overline{\mathbb{R}}$ accumulation point of I, $f : I \to \mathbb{R}$ function. We define*

$$\limsup_{x \to x_0} f(x) = \inf_{U \, neighbourhood \, of \, x_0} \, \sup_{x \in U \cap I} f(x) \qquad (6.39)$$

$$\liminf_{x \to x_0} f(x) = \sup_{U \, neighbourhood \, of \, x_0} \, \inf_{x \in U \cap I} f(x) \qquad (6.40)$$

where the first "inf" (resp. the "sup") is performed with respect to the family of all the deleted neighbourhoods U of x_0; and the neighbourhoods will be right or left neighbourhoods if the limit is from right or left.