Remark 6.41. [20G](Solved on 2022-11-29) *Using the properties of* inf, sup, *we obtain for example these characterizations*

$$\limsup_{x \to x_0} f(x) \le l \iff \forall z > l, \text{ eventually for } x \to x_0, f(x) < z \ ;$$

$$\limsup_{x \to x_0} f(x) \ge l \iff \forall z < l, \text{frequently for } x \to x_0, f(x) > z \ ;$$

and so on. (In this simplified writing, we assume that $x \in I$).