

Definition 6.26. [20H] (Solved on 2022-11-24) Given J an index set (not empty), let $a_n \in \mathbb{R}$ for $n \in J$. The supremum and infimum are defined as

$$\sup_{n \in J} a_n = \sup A \quad , \quad \inf_{n \in J} a_n = \inf A$$

where $A = \{a_n : n \in J\}$ is the image of the sequence.

Given D not empty, let $f : D \rightarrow \mathbb{R}$ be a function. The supremum and infimum are defined as

$$\sup_{x \in D} f(x) = \sup A \quad , \quad \inf_{x \in D} f(x) = \inf A$$

where $A = \{f(x) : x \in D\}$ is the image of the function.