

Corollary 6.42. [20N] You have $\alpha = \limsup_{x \rightarrow x_0} f(x)$ if and only if

$\forall z > \alpha$, eventually for $x \rightarrow x_0$, $f(x) < z$;

$\forall z < \alpha$, frequently for $x \rightarrow x_0$, $f(x) > z$.