Exercise 5.8. [20T] Prerequisites: [202]. *Given $\alpha \neq 0$ in a field, define that $\alpha^0 = 1$ and let α^{-n} be the multiplicative inverse of α^n when $n \geq 1$ natural. (Use [202]). For $n, m \in \mathbb{Z}$ show that*

$$\alpha^n \alpha^m = \alpha^{n+m} \quad , \quad (\alpha^h)^k = \alpha^{(hk)} \quad ;$$

if the field is ordered and $\alpha > 1$ show that $n \mapsto \alpha^n$ is strictly monotonic increasing.