Exercise 6.8. [20V] Prerequisites: [20T]. *Let $\alpha > 0, \alpha \in \mathbb{R}$ be fixed. We know that, for every natural $n \geq 1$, there exists an unique $\beta > 0$ such that $\beta^n = \alpha$, and β is denoted by the notation $\sqrt[n]{\alpha}$. (See e.g. Proposition 2.6.6 Chap. 2 Sec. 6 of the course notes [**?**] or Theorem 1.21 in [**?**]). Given $q \in \mathbb{Q}$, we write $q = n/m$ with $n, m \in \mathbb{Z}, m \geq 1$, we define*

$$\alpha^q \stackrel{def}{=} \sqrt[m]{\alpha^n} \quad .$$

Show that this definition does not depend on the choice of representation $q = n/m$; that

$$\alpha^q = \left(\sqrt[m]{\alpha}\right)^n \quad ;$$

that for $p, q \in \mathbb{Q}$

$$\alpha^q \alpha^p = \alpha^{p+q} \quad , \quad (\alpha^p)^q = \alpha^{(pq)} \quad ;$$

show that when $\alpha > 1$ then $p \mapsto \alpha^p$ is strictly monotonic increasing.