

Exercise 6.26. [20Y] (Proposed on 2022-12) Calculate supremum and infimum of the following sets (where n, m are integers).

$$\left\{ \frac{mn}{m^2 + n^2} : n, m \geq 1 \right\}, \quad \left\{ \frac{mn}{m + n} : n, m \geq 1 \right\}$$
$$\{2^n + 2^m : n, m \in \mathbb{N}\}, \quad \{2^n + 2^m : n, m \in \mathbb{Z}\}$$

$$\left\{ \frac{m^2 - 2}{n} : n, m \in \mathbb{Z}, n \neq 0 \right\}, \quad \left\{ \frac{m + 1}{m^2} : m \in \mathbb{Z}, m \neq 0 \right\}$$