Proof. [21B]

- If $\alpha < 1$, having fixed $L \in (\alpha, 1)$ you have eventually $\sqrt[n]{|a_n|} < L$ so there is a N for which $|a_n| \leq L^{N-n}$ for each $n \geq N$ and we conclude by comparison with the geometric series.
- For the two series $1/n$ and $1/n^2$ you have $\alpha = 1$.
- If $\alpha > 1$ you have frequently $\sqrt[n]{|a_n|} > 1$ So $|a_n| > 1$, contrary to the necessary criterion.

□