

Theorem 7.25. [21F] Let $\{a_n\}$ and $\{b_n\}$ be two sequences. If b_n tends monotonically to 0 and if the series of partial sums of a_n is bounded, i.e. if

$$b_n \geq b_{n+1} > 0 \quad , \quad \lim_{n \rightarrow \infty} b_n = 0 \quad , \quad \exists M > 0, \forall N \in \mathbb{N}, \left| \sum_{n=1}^N a_n \right| < M$$

then the series

$$\sum_{n=1}^{+\infty} a_n b_n$$

is convergent.

The proof is left as an exercise (Hint: use [21H])

Solution 1. [21G]