Definition 3.137. [21W]*Given two ordered sets $(X, \leq_X)$ and $(Y, \leq_Y)$, with X, Y disjoint,* ***the concatenation of*** *X* ***with*** *Y is obtained defining $Z = X \cup Y$ and providing it with the ordering $\leq_Z$ given by:*

- *if $z_1, z_2 \in X$ then $z_1 \leq_Z z_2$ if and only if $z_1 \leq_X z_2$;*
- *if $z_1, z_2 \in Y$ then $z_1 \leq_Z z_2$ if and only if $z_1 \leq_Y z_2$;*
- *If $z_1 \in X$ and $z_2 \in Y$ then you always have $z_1 \leq_Z z_2$.*

This operation is sometimes denoted by the notation $Z = X \mathbin{+\!\!\!+} Y$.
If the sets are not disjoint, we can replace them with disjoint sets defined by $\tilde{X} = \{0\} \times X$ and $\tilde{Y} = \{1\} \times Y$, then we may "copy" the respective orders, and finally we can perform the concatenation of $\tilde{X}$ and $\tilde{Y}$.