Exercises

3.211 [22F]Prerequisites:[21W],[071],[07V],[07X].Difficulty:*.Note:*exercise* 2

written exam on 29 January 2021.(Solved on 2022-10-13

in part)

Let be given $(X, \leq_X)$ where X is an infinite set and $\leq_X$ is a well ordering.

- If X has no maximum, then there exists $(Y, \leq_Y)$ such that setting $Z = Y \times \mathbb{N}$ with $\leq_Z$ the lexicographical order, then $(X, \leq_X)$ and $(Z, \leq_Z)$ have the same type of order.
- If instead X has maximum, then there exist $(Y, \leq_Y)$ and $k \in \mathbb{N}$ such that, setting Z be the concatenation of $Y \times \mathbb{N}$ and $\{0, \dots k\}$ (where $Y \times \mathbb{N}$ has the lexicographical order, as above), then $(X, \leq_X)$ and $(Z, \leq_Z)$ have the same type of order.
- Show that, in the previous cases, Y is well ordered.

Solution 1. *[22G]*