Lemma 3.120. [22S]*Let* $A \subseteq X$ *be a not empty set. We recall these properties of the supremum.*

1. *If* A *has maximum* m *then* $m = \sup A$.

2. *Let* $s \in X$. *We have* $s = \sup A$ *if and only if*

   - *for every* $x \in A$ *we have* $x \leq s$.
   - *for every* $x \in X$ *with* $x < s$ *there exists* $y \in A$ *with* $x < y$.

This last property is of very wide use in the analysis!
The proof is left as a (useful) exercise.

Solution 1. [22T]