Theorem 3.183. [23B] *Let $A \supseteq \mathbb{N}$ and $P(n)$ be a logical proposition that can be evaluated for $n \in A$. Suppose the following two assumptions are satisfied:*

- *$P(n)$ is true for $n = 0$ and*
- *$\forall n \in \mathbb{N}, P(n) \Rightarrow P(S(n))$;*

then P is true for every $n \in \mathbb{N}$.
The first hypothesis is known as "the basis of induction" and the second as "inductive step"

Proof. Let $U = \{n \in \mathbb{N} : P(n)\}$, we know that $0 \in U$ and that $\forall x, x \in U \Rightarrow S(x) \in U$, so U is S-saturated and $U \subseteq N$ we conclude that $U = \mathbb{N}$. □