Definition 23.20. [23Y]

- *Suppose the curves in the plane are described by the equation in implicit form* $F(x, y, a) = 0$*; that is, fixed the parameter* a*, the curve is the locus*

$$\{(x, y) : F(x, y, a) = 0\} \quad ;$$

  Then the envelope is obtained by expliciting the variable a *from the equation* $\frac{\partial}{\partial a} F(x, y, a) = 0$ *and substituting it into the* $F(x, y, a) = 0$.

- *For simplicity, consider curves that are functions of the abscissa. Let* $y = f(x, a) = f_a(x)$ *be a family of functions, with* $x \in I, a \in J$ *(open intervals), then* $y = g(x)$ ***is the envelope of*** f_a *if the graph of* g *is covered by the union of the graphs of* f_a *and the curve* g *is tangent to every* f_a *where it touches it. More precisely, for every* $x \in I$ *there is* $a \in J$ *for which* $g(x) = f(x, a)$*, and also, for every choice of* a *that satisfies* $g(x) = f(x, a)$*, we have* $g'(x) = f'(x, a)$.