3.2.1 Naive set theory [242]

As already explained in Definition [1X2], in set theory, the connective ”$\in$” is added; given two sets z, y the formula $x \in y$ reads ”*x belongs to y*” or more simply ”*x is in y*”, and indicates that x is an element of y.

It is customary to indicate the sets using capitalized letters as variables.

Definition 3.36. *[1Y8]*

[226]

Definition 3.37. *[227]*

It is usual to write $x \notin y$ for $\neg(x \in y)$, $x \nsubseteq y$ for $\neg(x \subseteq y)$ and so on.

Remark 3.38. *[1W0]*

We also define the constant $\emptyset$, also referred to as $\{\}$, which is the empty set,[14] that is uniquely identified by the property

$$\forall x, \neg x \in \emptyset \quad .$$

Some fundamental concepts are therefore introduced: union, intersection, symmetric difference, power set, Cartesian product, relations, functions *etc*.

Definition 3.39. *[1Y2]*

Definition 3.40. *[1W1]*

The power set is defined as in [1Y1].

Definition 3.41. *[23S]*

Exercises

E3.42 [1W6]

E3.43 [1W8]

E3.44 [1W9]

E3.45 [1WB]

E3.46 [1W2]

E3.47 [1WF]

E3.48 [1Y4]

E3.49 [1WC]

E3.50 [24P]

Remark 3.51. *[01J]*

While in formal theory all the elements of language are sets, in practice we tend to distinguish between the sets, and other objects of Mathematics (numbers, functions, etc etc); for this in the following we will generally use capital letters to indicate the sets, and lowercase letters to indicate other objects.

[14] In Zermelo–Fraenkel axiomatic theory, the existence of $\emptyset$ is an axiom.