3.8 Natural numbers in ZF [246]

In this section we will build a model of the natural numbers inside the ZF set theory; this model satisfies Peano's axioms [1XD] and has an order relation that satisfies [26H], so this model enjoys all properties described in Sec. [1X9]; for this reason in this section we will mostly discuss properties that are specific of this model.

3.8.1 Successor

Definition 3.195. *[24X]*

Exercises

E3.196 [24V]

E3.197 [24M]

E3.198 [239]

E3.199 [245]

E3.200 [1YM]

E3.201 [24Q]

E3.202 [24S]

3.8.2 Natural numbers in ZF

Definition 3.203. *[243]*

Using the axiom of infinity [243] we can prove the existence of the set of natural numbers.

Theorem 3.204. *[244]*

Example 3.205. *[291]*

Remark 3.206. *[25C]*

We can also prove directly the induction principle.

Theorem 3.207 (Induction Principle)**.** *[23B]*

Theorem 3.208. *[24D]*

To prove the above theorem, the exercises in the following section can be used.

Remark 3.209. *[26K]*

The ordered set $\mathbb{N}, \subseteq$ then enjoys these properties.

Proposition 3.210. *[26J]*

More details are in the course notes (Chap. 1 Sec. 7 in [**?**]); or [**?**],[**?**].

3.8.3 Transitive sets

Definition 3.211. *[24Z]*

Example 3.212. *[290]*

Exercises

E3.213 [25J]

E3.214 [257]

E3.215 [26N]

E3.216 [26P]

The previous exercises prove Theorem [24D], then by results of Sec. [1X9] we obtain that $(\mathbb{N}, \leq)$ is well ordered.

Here following are other interesting exercises.

Exercises

E3.217 [269]

E3.218 [265]

E3.219 [25D]

E3.220 [25W]

E3.221 [25Z]

3.8.4 Ordinals

Perusing the above results we can give some elements of the theory of ordinals.

Definition 3.222. *[26D]*

Exercises

E3.223 [25Q]

E3.224 [25B]

E3.225 [25N]

E3.226 [25M]

E3.227 [25G]

E3.228 [255]

E3.229 [26S]

E3.230 [26V]

Remark 3.231. *[275]*

[(da sistemare)]