

Remark 3.h.15. [25C] *This fact holds true:*

$$\forall y \in \mathbb{N}, y \neq \emptyset \Rightarrow \exists x \in \mathbb{N}, S(x) = y$$

this can be proven by induction, as in [1YP], or by proving that, if

$$\exists y \in \mathbb{N}, y \neq \emptyset \wedge \forall x \in \mathbb{N}, S(x) \neq y$$

then $\mathbb{N} \setminus \{y\}$ would be an S -saturated set smaller than \mathbb{N} , a contradiction. In particular by [1YM] we get that the successor function

$$S : \mathbb{N} \rightarrow \mathbb{N} \setminus \{0\}$$

is bijective.

*If $n \neq 0$, we will call $S^{-1}(n)$ the **predecessor** of n .*