

## 4.5 Z-F and Peano compatibility

[26F]

Let's go back now to the model  $\mathbb{N}_{ZF}$  of  $\mathbb{N}$  built relying on the theory of Zermelo—Fraenkel, seen in Sec. [246]. We want to see that this model satisfies Peano's axioms.

Recall that, given  $x$  (any set, not necessarily natural number) the successor is defined as

$$S(x) \stackrel{\text{def}}{=} x \cup \{x\} .$$

It's easy to see that **N1** and **N3** are true. The **N5** property follows from the fact that  $\mathbb{N}_{ZF}$  is the smallest set that is S-saturated. **N2** and **N4**, derive from [1YM].

We moreover saw in Theorem [24D] that the relation  $\subseteq$  satisfies the requisites of Hypothesis [26H].