Remark 3.209. [275]*Consider again Proposition* [26J] *that states that* $\mathbb{N}_{ZF}$ *is well ordered by the relation* $\subseteq$.
We know by [255] *and* [257] *that* $\mathbb{N}_{ZF}$ *is an ordinal; we may be tempted to see Proposition* [26J] *as a corollary of the previous result* [26V].
This is unfortunately not a well posed way of proving this result, because of this cascade of dependencies:

- *the proof of* [26V] *relies on the result* [263]
- *the result* [263] *in turn needs a definition by recurrence of a function: this is Theorem* [08Z]
- *the proof of Theorem* [08Z] *uses the fact that the induction principle holds on* $\mathbb{N}$.

So we need to first prove the properties of $\mathbb{N}_{ZF}$ *independently of the theory of ordinals, and then prove the results in Sec.* [1X9], *and then eventually we can prove the result* [26V], *that states that any ordinal is well ordered by the relation* $\subseteq$.