Proposition 4.13. [27P](Replaces 27Y) *Addition is commutative.*

Proof. By the lemma we can write

$$S(h) + n = S(h + n) = h + S(n) \qquad (4.14)$$

intuitively the formula is symmetric and therefore also the definition of addition must have a symmetry. Precisely, let $\tilde{f}_n(h) \stackrel{\text{def}}{=} f_h(n)$ then $\tilde{f}_n(0) = n$ (as already noted) and for the lemma [27N] $\tilde{f}_n(S(h)) = S\Big(\tilde{f}_n(h)\Big)$ but then $\tilde{f}$ satisfies the same recursive relation as f and therefore they are identical, so $f_h(n) = f_n(h)$. (The idea is that if we had defined addition. recursively starting from left instead of right, we would have achieved the same result). □