Exercises

E0.1 [27Y]

E0.2 ★ Show that addition is associative.

Solution 1. *We want to show that*

$$\forall n, m, k \in \mathbb{N} \quad , \quad (n + m) + h = n + (m + h) \quad ;$$

Define

$$P(n) \doteq \forall m, k \in \mathbb{N} \quad , \quad (n + m) + h = n + (m + h) \quad ;$$

Obviously $P(0)$ *is true; Study*

$$P(S(n)) \doteq \forall m, k \in \mathbb{N} \quad , \quad (S(n) + m) + h \stackrel{?}{=} S(n) + (m + h) \quad ;$$

we use the [(4.15)]

$$S(h) + n = S(h + n) = h + S(n)$$

Write

$$(S(n) + m) + h \stackrel{\text{per [(4.15)]}}{=} S\Big((n + m) + h\Big) \stackrel{P(n)}{=} S\Big(n + (m + h)\Big) \stackrel{p}{}$$

thus concluding the inductive step.