Proposition 4.40. *[28Z] $\leq$ is a total order relation.*

Proof. Consider the proposition

$$P(n) \doteq \forall m \in \mathbb{N}, n \leq m \vee m \leq n$$

then $P(0)$ is true. Let's assume $P(n)$; let's fix an m;

- if $m \leq n$ then $m \leq S(n)$, by the lemma (point [2]), so $P(Sn)$ holds;
- if $\neg m \leq n$ but $P(n)$ holds, then $n \leq m$ must hold, but it cannot be $n = m$, so $n < m$ holds: but then $S(n) \leq m$ by the lemma (point [4]);

in any case $P(S(n))$ is proven starting from $P(n)$. □