Exercise 6.48. *[29R]Prerequisites:[20F],[29J],[06M],[0FR],[0FT].Difficulty:**

on 2022-11-24)

Let $I \subset \mathbb{R}$, $x_0 \in \overline{\mathbb{R}}$ accumulation point of I, $f : I \to \mathbb{R}$ function. As in [29J] $\mathcal{F}$ all the neighbourhoods of x_0 with associated the filtering ordering

$$U, V \in \mathcal{F} \quad , U \leq V \iff U \supseteq V \quad .$$

Let

$$s, i : \mathcal{F} \to \mathbb{R} \quad , \quad s(U) = \sup_{x \in U \cap I} f(x) \quad , \quad i(U) = \inf_{x \in U \cap I} f(x)$$

note that these are monotonic functions, and show that [a]

$$\limsup_{x \to x_0} f(x) \stackrel{def}{=} \inf_{U \in \mathcal{F}} s(U) = \lim_{U \in \mathcal{F}} s(U) \tag{6.49}$$

$$\liminf_{x \to x_0} f(x) \stackrel{def}{=} \sup_{U \in \mathcal{F}} i(U) = \lim_{U \in \mathcal{F}} i(U) \tag{6.50}$$

where the limits are defined in [0FR].

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[a] *cf* [6.39], [(6.40)].