Exercises

E3.d.33 [2B2] Suppose that on the set X there is a relation R that is reflexive and transitive and satisfies

$$\forall x, y \in X \; \exists z \in X, \; xRz, yRz \quad . \tag{3.d.34}$$

(as seen in [(3.96)])

This pair (X, R) is a "Directed Set" according to the usual definition (see [17] or other references in [43]).

Show that there exists another relation $\leq$ such that

- $\leq$ is a partial order and it satisfies [(3.96)];
- R extends $\leq$ that is;

$$\forall x, y \in X \; x \leq y \Rightarrow \; xRy \quad ;$$

- moreover $(X, \leq)$ is cofinal in (X, R).

Solution 1. *[2GM]*